Non-Boolean Conditionals

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1. Overview. Standard theories predict that indicative conditionals (ICs) behave in a Boolean fashion when interacting with *and* and *or*. We test this prediction by investigating probability judgments about sentences of the form $\ulcorner a \to b \{ \text{ AND/OR } \} c \to d \urcorner$. Our findings are incompatible with a Boolean picture. This is challenging for truth-conditional theories of ICs, as well as for several other theories. Some trivalent theories hold promise for providing an account of our data.

2. Background. Boolean interpretations of *and* and *or* entail constraints about probabilities of compounds (see e.g. Adams 1998). The following two are relevant here:

***and*-drop.** If $\mathrm{A} \not\models \mathrm{B}$, $Pr(\mathrm{A}) > Pr(\mathrm{A} \wedge \mathrm{B})$ ***or*-drop.** If $\mathrm{A} \not\models \mathrm{B}$, $Pr(\mathrm{A} \vee \mathrm{B}) > Pr(\mathrm{A})$

These constraints apply to all sentences of natural language that express propositions. Thus, if truth-conditional theories of ICs are correct (see a.o. Stalnaker 1968, Kratzer 2012), the sentences in (1) are predicted to conform to the constraints on the right below.

(1) a. If Lea danced, Mia danced, or, If Lea didn't dance, Nina danced.
b. If Lea danced, Mia danced.
c. If Lea danced, Mia danced, and, If Lea didn't dance, Nina danced.

$$Pr(1a) > Pr(1b)$$
$$Pr(1b) > Pr(1c)$$

3. Experiment 1. Our experiment tests ***and*-drop** and ***or*-drop** for natural language ICs. Rather than relying on assumptions about probabilities of conditionals (like Stalnaker's Thesis; see Stalnaker 1970), subjects were asked to assign probabilities on the basis of observed frequencies.

After an exposition period (Experience phase), subjects were presented with several sentences and asked to perform a likelihood estimation task (Test Phase). Three main variables were manipulated: presence and type of connective (And vs Or vs None; within); compatibility of the two antecedents, when sentences involved two ICs (Compatible vs Incompatible; between); and frequency of the event described in the consequent, given the antecedent (50/50 vs 75/25; between).

In Experience, participants viewed 24 animations of 1 shape (Incompatible conditions) or 1-2 shapes (Compatible) traveling by "car" into a "tunnel", whereupon they changed into 1 of 2 colors (**Fig.1**). Then, participants answered (2), and were included only if they answered "yes" to both ($N = 153$). In Test, participants viewed two sets of 4 "mystery car" animations, and gave likelihood estimates for (i) the simple ICs in (3) and (ii) the compounds schematized in (4).

(2) a. If the SQUARE enters the tunnel, it always turns RED or YELLOW.
b. If the CIRCLE enters the tunnel, it always turns GREEN or BLUE.

(3) a. If the car was carrying the SQUARE, the SQUARE turned { RED / YELLOW }. $s \to r$, $s \to y$
b. If the car was carrying the CIRCLE, the CIRCLE turned { GREEN / BLUE }. $c \to g$, $c \to b$

(4) a. $s \to r \{ \text{ AND / OR } \} c \to g$
b. $s \to y \{ \text{ AND / OR } \} c \to b$

Findings. Our participants overestimated input frequencies in the 50/50 condition ('balanced' inputs occurred 50% of the time, mean estimate 68%) and in the lower frequency events of the 75/25 condition ('lower' input 25%, estimate 46%; cp. 'higher' input 75%, estimate 75%). Importantly for us, the ordering between estimates was accurate, and they were significantly different,

$F(1,148) = 8.15, p < .005$.[1] Also, and crucially, likelihood estimates were *not* impacted by the factors Compatibility or Connective, ps $> .53$. See **Fig.2** (L).

Discussion. ***and*-drop** or ***or*-drop** predict lower probability estimates for $\ulcorner s \to r$ OR $c \to g \urcorner$ over $\ulcorner s \to r \urcorner$, and for $\ulcorner s \to r \urcorner$ over $\ulcorner s \to r$ AND $c \to g \urcorner$. This asymmetry was not observed, revealing non-Boolean behavior. The contrast between estimated probabilities in the 50/50 and 75/25 conditions shows that subjects did make discriminating probabilistic judgments.

4. Analysis Our findings are challenging for all theories that vindicate ***and*-drop** and ***or*-drop**. Conversely, they can be explained by some trivalent theories (in particular Bradley 2002; see Rothschild 2014, Lassiter 2019 a.o. for similar views). Every clause A has definedness conditions $\mathrm{D}(\mathrm{A})$ and truth conditions $\mathrm{T}(\mathrm{A})$. $\mathrm{A} \to \mathrm{B}$ is defined iff A is true and B is defined, and true iff A and B are true. $\mathrm{A} \wedge \mathrm{B}$ ($\mathrm{A} \vee \mathrm{B}$) is defined iff at least one of A and B is defined, and true iff all (at least one of) the defined conjuncts (disjuncts) are true.

$$[\![\mathrm{A} \to \mathrm{B}]\!] = \begin{cases} \text{def. at } w \text{ iff } w \in \mathrm{T}(\mathrm{A}) \text{ and } w \in \mathrm{D}(\mathrm{B}) \\ \text{true at } w \text{ iff } w \in \mathrm{T}(\mathrm{A}) \cap \mathrm{T}(\mathrm{B}) \end{cases} \qquad [\![\mathrm{A} \wedge (\vee) \mathrm{B}]\!] = \begin{cases} \text{def. at } w \text{ iff } w \in \mathrm{D}(\mathrm{A}) \text{ or } w \in \mathrm{D}(\mathrm{B}) \\ \text{true at } w \text{ iff: if } w \in \mathrm{D}(\mathrm{A}),\ w \in \mathrm{T}(\mathrm{A}) \\ \quad \text{and (or) if } w \in \mathrm{D}(\mathrm{B}),\ w \in \mathrm{T}(\mathrm{B}) \end{cases}$$

Combined with a notion of trivalent probability (see Cantwell 2006), this semantics predicts failures of ***and*-drop** and ***or*-drop**.

5. Experiment 2 (control). One could worry that our findings reflect a flawed novel experimental paradigm. In response, we tested non-conditional sentences. We replaced the sentences in (3) with those in (5), modified the Test animations so that the mystery car initially shows the two shapes, and replaced the sentences in (4) with conjunctions/disjunctions of (5a) and (5b).

(5) a. The SQUARE turned { RED / YELLOW }. r, y

b. The CIRCLE turned { GREEN / BLUE }. g, b

Findings (n=83). We found a main effect of Connective in Experiment 2, $p < 0.0001$, due to estimates for *and* differing significantly from *or* and none (*and* 58.5%, *or* 68.6%, none 66.1%) , both ps < 0.007. This shows expected Boolean behavior at least in the 50/50 condition, alleviating concerns that our paradigm wouldn't be sensitive enough to detect such behavior. See **Fig.2** (R).

Fig.1: (a) 1-, 2-traveler scenes, (b) mystery scene, (c) Incompatible & (d) Compatible event schemas.

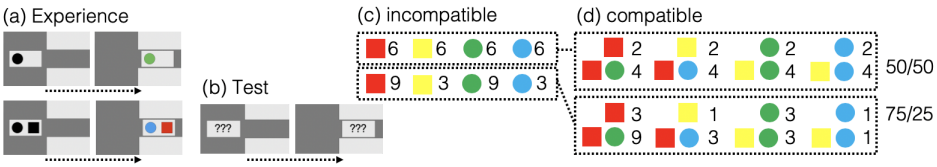
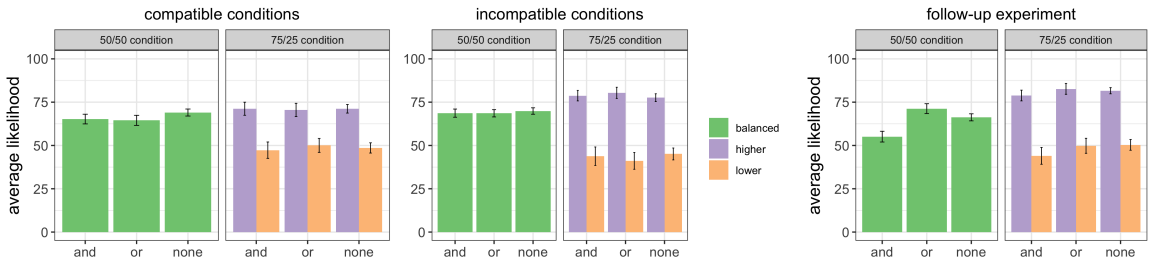


Fig.2: Results for Experiment 1 (L) and Experiment 2 (R).



[1] We report the results of a 3x2x2 ANOVA with a within-subject error term for connective type.